

Multi-objective optimization of GPU3 Stirling engine using third order analysis



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ARTICLE INFO

Article history:

Received 4 February 2014

Accepted 19 June 2014

Available online 7 August 2014

Keywords:

Stirling engine

Optimization

Third order thermodynamics

GPU3

ABSTRACT

Stirling engine is an external combustion engine that uses any external heat source to generate mechanical power which operates at closed cycles. These engines are good choices for using in power generation systems; because these engines present a reasonable theoretical efficiency which can be closer to the Carnot efficiency, comparing with other reciprocating thermal engines. Hence, many studies have been conducted on Stirling engines and the third order thermodynamic analysis is one of them. In this study, multi-objective optimization with four decision variables including the temperature of heat source, stroke, mean effective pressure, and the engine frequency were applied in order to increase the efficiency and output power and reduce the pressure drop. Three decision-making procedures were applied to optimize the answers from the results. At last, the applied methods were compared with the results obtained of one experimental work and a good agreement was observed.

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1. Introduction

According to the growing enhancement of fossil fuels consumption and limited energy resources in the world, the environmental pollution, and global warming, the attentions have been focused on the field of energy, especially renewable energy, in recent years. Solar energy is one of the most important sources of renewable energies, and Stirling engine is one of the functional techniques that uses solar energy [1,2]. Stirling engine is an external heat engine that works at closed cycles. The heat from the external heat source is transferred to working fluid and is converted to work via the expansion of the working fluid within the cylinder. Since Stirling engine is an external heat engine, a variety of thermal resources like solar energy, nuclear energy, the heat produced from the combustion of fossil fuels and waste heat from industrial processes can be used as the external heat source [3]. High thermal efficiency, minimal pollution, reliability, maximum utilization and clean combustion are the major features of Stirling engines. Due to the potential of Stirling engine which was mentioned above, this engine is a good alternative to the conventional power generation systems. Hence, many studies have been carried out on the modeling of Stirling engines of which, some are mentioned in the following sections.

The first analysis on the performance of Stirling engines was presented by Schmidt in 1871. The Schmidt's theory is an effective tool in the design of Stirling engines [4]. The assumptions that Schmidt considered in his analysis included: There is no temperature gradient in heat exchangers, the surface temperature of the cylinder and the piston is constant, the mass of the fluid is constant which implies that there is no leakage, the perfect gas equation of state is applied, the volume changes are sinusoidal, and the speed of working fluid within the engine is considered constant. One of the aforementioned assumptions was that the gas behaves isothermal in the expansion and compression spaces. In practice, this assumption is not true, especially at high speeds. At the higher speeds, the expansion and compression processes are closer to adiabatic process. Based on this fact, Finkelstein presented adiabatic model (the second order model) for creating a more realistic analysis of Stirling engine [5]. In adiabatic model, the engine is divided to five parts (cooler, heater, regenerator, expansion and compression spaces). Each part of the engine is considered as a control volume and the conservation laws of mass and energy are analyzed for each part [5].

Another analysis of Stirling engine is the third order analysis. In the third order model, the equations of mass, momentum and energy of engine are written in the form of partial differential equations for several control volumes. Some examples of such models were investigated by Dyson et al. [4] and Anderson [6]. Firstly, this model was applied by Feurer for designing Stirling

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Nomenclature

d	displacer stroke (cm)	v_f	gas velocity (m/s)
r_c	crank radius (m)	ρ	gas density (kg/m ³)
A_f	gas flow area (m ²)	n	normal vector on control-volume surface
f	engine frequency (Hz)	x	control-volume length (m)
η	engine efficiency	A_w	wall area of control-volume (m ²)
p	engine output power (W)	S	connecting surfaces between adjacent control-volumes (m ²)
Q_r	total heat rate rejected from the engine (W)	V	volume of control-volume (m ³)
T_c	heat rejection temperature (K)	T	gas temperature inside control-volume (K)
T_e	heat absorption temperature (K)	P	gas pressure inside control-volume (Pa)
Re	Reynolds number of control-volume gas	τ_f	gas shear stress from control-volume wall (Pa)
Re_w	Reynolds number of control-volume gas	u	gas specific internal energy (J/kg)
μ	dynamic viscosity of control-volume gas (Pa s)	Q_c	net absorbed heat of control-volume gas (J)
Nu	Nusselt number	Q_e	net injected heat to control-volume (J)
k	control volume index	m_w	mass of control-volume wall (kg)
K	Working gas conductivity (W/m K)	c_w	specific heat of control-volume wall (kJ/kg)
t	time (s)	T_w	temperature of control-volume wall (K)
d_{i+}	distance from the positive ideal point	r	working gas constant (kJ/kg K)
d_{i-}	distance from the negative ideal point	T_H	temperature of the heat source (K)
C_v	specific heat at constant volume (kJ/kg K)	V_{cle}	expansion dead volume
L	length of connecting rod (m)	A_d	sectional area of the displacer piston (m ²)
e	eccentricity	A_p	sectional area of the power piston (m ²)
V_{clc}	compression dead volume	dP	Pressure drop (bar)
D_h	hydraulic diameter (m)		

engine [7]. The third order analysis is used for the optimization methods especially for evaluating the efficiency, output power, and the behavior of the working fluid flow inside the engine. So far, several algorithms have been used for optimization of Stirling engines. Each of these algorithms is attended on the part of the predicted results by a mathematical model. By recording the changes of the results, the orientation of the process optimization and the optimal values of the design variables were determined. For example, Carlsen by using two variables power and thermal efficiency that was obtained from a mathematical model increased the efficiency of Stirling engine up to 23% [8]. Chen used genetic algorithms to optimize a solar Stirling engine and the results of the optimization were implemented on the engine [9].

Kraitong and Mahkamov carried out theoretical investigations on the determination of optimal design parameters of a low temperature difference (LTD) solar Stirling engine using the optimization method based on Genetic algorithms [10]. For considering all the aforementioned issues, we followed the third-order model and studied multi-objective functions including the pressure drop (bar), the output power (kW) and the efficiency of the considered Stirling engine.

The multi-objective optimization has been applied by different engineering problems such as skyline computation and vehicle routing issues [10–12]. Evolutionary algorithms (EA) were applied in an attempt to stochastically solve problems of this generic class during the 18th century [13]. Inquiring a set of solutions, of which each satisfies the objectives at an acceptable level without being overcome by any other solution, is a reasonable solving method to a multi-objective quandary [14]. Multi-objective optimization of various thermodynamic and energy systems have been of high interest attention of researchers these days [15–21].

In this study, the multi-objective optimization is conducted with four decision variables including the temperature of the heat source, the engine stroke, the mean effective pressure, and the engine frequency in order to increase the efficiency and power and reduce the pressure drop. Here, both second-order and third-order optimizations are applied so the parameters were considered according to the following parameter package order:

- Efficiency and pressure drop.
- Power and pressure drop.
- Power, efficiency and pressure drop.

2. Stirling system

Stirling engine is a type of hot air engine which can produce mechanical work using heat transfer between the hot and cold sources. In the Stirling engine working cycle, the external heat is transferred to the working fluid at the maximum temperature of the cycle and the heat is rejected from the working fluid at the lowest temperature of the cycle.

When the working fluid passes through the regenerator during the movement from the hot side to the cold side of the engine, some heat is absorbed by the regenerator. This heat is stored in the regenerator, and rejected to the working fluid with minimal thermal dissipation while the working fluid passes from the cold space to the hot space. While displacer moves reciprocating, it forces the working fluid to a reciprocating movement by passing through the regenerator between the hot side and the cold side [22].

Various types of Stirling engines include: Gamma type Stirling engine, Beta type Stirling engine and Alpha type Stirling engine [24–26]. All of them have the same thermodynamic cycle, but the mechanical mechanisms of them are different. The Beta Stirling engine is the oldest structure of Stirling engine (Fig. 1). These types of engines use the combination of power piston and displacer [27]. The construction of this engine is in such a way that both pistons are located linearly in a cylinder. The displacer piston moves the working fluid among the heater, the regenerator, and the cooler between the hot and cold spaces [28]. The studied engine for this research was the GPU-3 whose characterizations are demonstrated in Table 1.

3. Optimization

Due to the necessity to optimize several objectives simultaneously and access to several conflicting criteria, commonly the

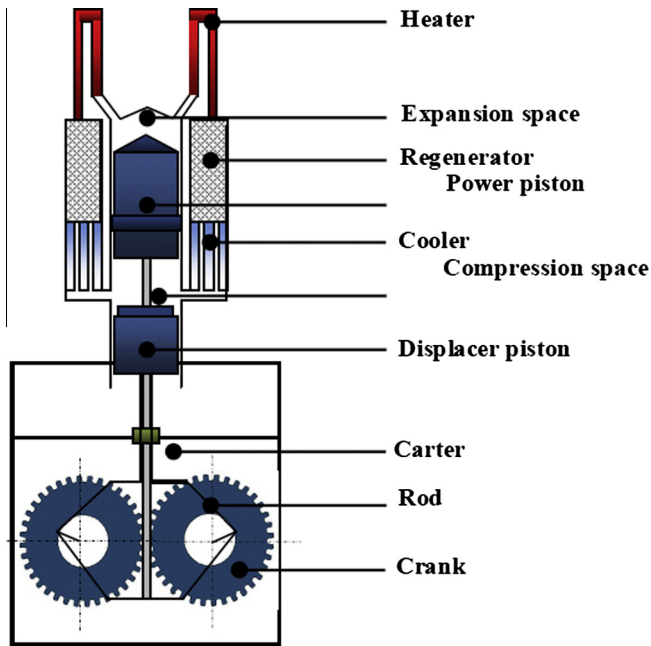


Fig. 1. Beta type Stirling engine [23].

Table 1
GPU-3 engine specification [23].

Specifications	Values
Type	Beta
Heat source T_H	977 K
Heat sink T_C	288 K
Mean pressure	4.13 MPa
Working fluid	Helium
Working frequency	41.7 Hz
Regenerator length	0.0226 m
Wire diameter	40 μm
Regenerator diameter	0.0226 m
Total mass of gas	0.001135 kg
Porosity	0.697
Displacer diameter	0.069
Number of regenerators per cylinder	8
Duct number of gauzes of the matrix	308

combination of goals are used to find out the solution of multi-objective problems, while these problems must be solved as multi-objective. Unlike the single-objective optimization which searches unit optimal solution; in the multi-objective problems, due to a conflict between the goals, there is no single optimal solution. Thus, the method should search several optimal solutions that are called the Pareto set [29].

There are several techniques for solving the multi-objective optimization problems, among which the Epsilon-Constraint Method, the Weighted Sum Method, and the Evolutionary Algorithm can be counted. In this study, the multi-objective evolutionary algorithm has been applied due to the ability to solve complex problems and provide the optimal exchange curves between the objectives. These models can easily solve the problems that do not adherence certain continuity, the decision spaces are not integrated, and their objective functions have random parameters. This model was proposed by Deb to solve the problems of the classic genetic algorithm [30]. The main problem of the previous multi-objective optimization includes a bulk of calculations per iteration that leads to increasing the run-time of modeling and do not maintain the appropriate number of optimal values during the implementation of the model [31].

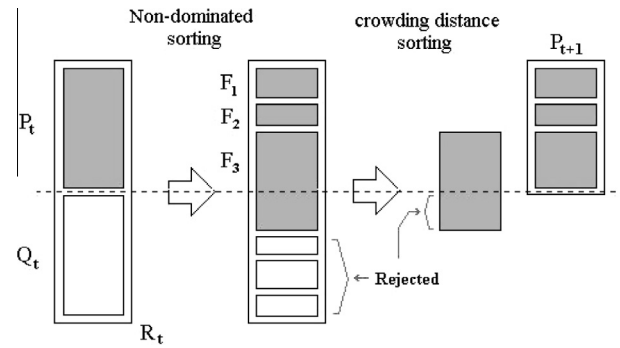


Fig. 2. The structure of NSGA-II multi-objective optimization model [32].

One of the ways that has shown its effectiveness in solving many multi-objective problems, and has been used for multi-objective optimization algorithm, is the NSGA-II optimization algorithm [30]. Fig. 2 shows the performance of NSGA-II multi-objective optimization model.

3.1. Decision making

For choosing the optimum point of the engine, a decision should be taken and there are different ways to make decisions. In this work, three methods including TOPSIS, LINMAP, and Fuzzy have been used which are explained briefly as followings. In the LINMAP method, the optimal point is a point on the Pareto Front that has the shortest distance to the ideal point. The ideal point is a point that has best situation. In the TOPSIS method, the optimal point is a point which has the lowest value of C_L .

$$C_L = \frac{d_i^-}{d_i^+ + d_i^-} \quad (1)$$

where d_i^+ and d_i^- are the distance of each point from the positive ideal point and negative ideal point, respectively. In the Fuzzy method, the Fuzzy Bellman–Zadeh is used which is explained in further details in the literature [33–35].

3.1.1. Bellman–Zadeh decision-making method

As the intersection of all fuzzy criteria and constraints, a final decision is defined by the Bellman and Zadeh model and this model is represented by its membership function. In this method, a matrix of membership function is assigned, where the fuzzy membership function is placed in the column for each objective. The rows of the membership matrix reflect the magnitudes of the membership functions for each solution of the Pareto frontier. Hence, the number of rows is equal to the number of the solutions and the number of columns in the membership function matrix is equal to the number of the objectives on the Pareto frontier. In the literature, the details of the method definition are described [33–35]. For the proposed solution, the fuzzy membership function of each solution is set at the minimum value of the membership functions of all objectives. Therefore, a vector of membership is obtained in which its elements are the minimum membership function of the objectives at each solution. As the final solution, the minimum distance with the ideal point is selected for the best solution. In other word, in the Bellman–Zadeh fuzzy decision making method, a maximum of minimums (minimum membership function of objectives) is selected as the final optimal solution. For more clarifications, the details are reported in other works [33–35].

3.2. Objective functions, decision variables and constraints

A mathematical model of the optimization process has three general parts (the objective functions, the decision variables and the constraints problem) that are described here in. Three objective functions are used for this study including the pressure drop inside the engine, the power and the efficiency of the Stirling engine. In this paper, four decision variables have been considered as followings:

- f : Engine's frequency.
- P : Mean effective pressure.
- d : Stroke.
- T_H : Temperature of heat source.

The objective functions with respect to the following constraints have been solved:

$$20 \leq f \leq 50 \text{ Hz} \quad (2)$$

$$2 \leq P \leq 5 \text{ MPa} \quad (3)$$

$$0.02 \leq d \leq 0.08 \text{ m} \quad (4)$$

$$800 \leq T_H \leq 1100 \text{ K} \quad (5)$$

4. Nlog analyzer

Nlog is a third order code in which, the engine is considered as a complex unit and the engine is divided into several parts. In order to achieve the heat transfer equations, the equations of mass, momentum and energy of the engine are written to describe the heat transfer processes for each part. In this method, the basic equations, in forms of partial differential equations, are written versus time and place. Then, some simplifications such as considering one-dimensional conditions for each part and steady flow are carried out [36]. Pressure drop and mechanical friction in the compression and expansion spaces are considered as losses. The following variables are entered into the Nlog code:

- The Stirling engine type.
- The geometric properties of all gas transferring channels, pipes, and chambers.
- The wetted wall surface areas and roughness factor for each.
- Regenerator's type, geometry and porosity.
- The properties of the non-ideal working gas (including compressibility factor, fluid thermal conductivity and viscosity relations).
- The engine initial charge pressure and local temperatures.
- The heat exchangers wall temperature gradient (constant along time period).
- Stiffness of springs if connected to any moving parts, neglecting spring mass.
- Geometries of the linkages between the engine moving parts (If exist).
- The external time varying force/torque applied to the engine.

The Nlog output consists of:

- The engine output power and efficiency.
- Pressure against total volume.
- Heat transfer rate against time.
- The working gas velocity in pipes and channels.
- Rigid bodies' velocity versus time.
- The temperature of working gas at any point of the engine versus time.

5. Theoretical model

In order to do conduct the optimization steps, the Nlog analyzer was applied while the code was modified with some complementary codes for the linkage between the analyzer and the optimization codes. Nlog is a third-order Stirling cycle analyzer which calculates the generated power and rejected heat by Stirling engine.

The code converts the whole gas transferring channels and pipes of the engine, to some divided control volumes and determines the gas dynamic and thermodynamic parameters for each control volume by solving the continuity, momentum and energy equations simultaneously. The standard forms of continuity, momentum and energy equations are presented by Eqs. (6)–(8), respectively [37].

$$\oint_{S_k} \rho v_f n dA_f + \frac{\partial}{\partial t} \left(\int_{V_k} \rho dV \right) = 0 \quad (6)$$

$$\oint_{S_k} \rho v_f n dA_f + \frac{\partial}{\partial t} \left(\int_{V_k} \rho dV \right) = 0 \quad (7)$$

$$\frac{\partial}{\partial t} \left(\int_{V_k} \rho v_f dV \right) + \int_{S_k} P n dA_f + \int_{S_k} (v_f \rho) v_f n dA_f + \tau_{fk} A_{wk} = 0$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\int_{V_k} \left(\rho u + \rho \frac{v_f^2}{2} \right) dV \right) + \int_{S_k} \left(\rho u + \rho \frac{v_f^2}{2} + P \right) v_f n dA_f \\ + \int_{S_k} P \frac{dx}{dt} n dA_r - \frac{dQ_c}{dt} = 0 \end{aligned} \quad (8)$$

The free flow area for the working gas transformation is assumed to be constant for each control volume, thus Eq. (6) is converted to Eq. (9). The area is also assumed to be constant during time, so Eq. (9) may be converted to Eq. (10) with this assumption.

$$\oint_{S_k} \rho v_f n dA_f + \frac{\partial}{\partial t} (\rho_k A_{fk} x_k) = 0 \quad (9)$$

$$\oint_{S_k} \rho v_f n dA_f + A_{fk} \frac{\partial}{\partial t} (\rho_k x_k) = 0 \quad (10)$$

The density and velocity of the working gas are assumed to be uniform over the cross sectional area between the adjacent control volumes and inside each control volume. Thus Eq. (11) becomes the final simplified continuity equation in the code.

$$\sum_{s=S_k} \rho_s v_{fs} n_s A_{fs} + A_{fk} \frac{\partial}{\partial t} (\rho_k x_k) = 0 \quad (11)$$

Extending the above assumptions of working gas velocity, density and cross sectional area, for working gas pressure in Eq. (7) will result in Eq. (12). The following is the simplified form of the momentum equation.

$$\frac{\partial}{\partial t} (\rho_k A_{fk} x_k v_{fk}) + \sum_{s=S_k} P_s n_s A_{fs} + \sum_{s=S_k} (v_{fs} \rho_s) v_{fs} n_s A_{fs} + \tau_{fk} A_{wk} = 0 \quad (12)$$

The energy equation is also simplified upon assuming the gas specific heat to be equal for every place of each control volume and is expressed as followings:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_k A_{fk} x_k \left(u_k + \frac{v_{fk}^2}{2} \right) \right) + \sum_{s=S_k} \left(\rho_s u_s + \rho_s \frac{v_{fs}^2}{2} + P_s \right) v_{fs} n_s A_{fs} \\ + A_{fk} P_k \frac{dx_k}{dt} - \frac{dQ_{ck}}{dt} = 0 \end{aligned} \quad (13)$$

The total heat transferred into each control volume can be calculated using the following relation:

$$\frac{dQ_{C_k}}{dt} = \frac{Nu_k K_k A_{w_k}}{D_{h_k}} (T_{w_k} - T_k) \quad (14)$$

The variation of wall temperature for each control volume is calculated from:

$$\frac{dT_{w_k}}{dt} + \frac{1}{C_w m_{w_k}} \left(\frac{dQ_{C_k}}{dt} - \frac{dQ_{e_k}}{dt} \right) = 0 \quad (15)$$

The gas specific internal energy is assumed to be only a function of gas temperature according to Eq. (16) for helium [38]:

$$u_k = C_v T_k, \quad (C_v = 3.11 \text{ kJ/kg K}) \quad (16)$$

The gas thermal conductivity is assumed to be a function of gas temperature and pressure according to Eq. (17) for helium [38].

$$K_k = 2.68 \times 10^{-3} \times \left(1 + 1.123 P_k 10^{-8} \right) \left(T_k^{71(1-2 \times 10^{-9} P_k)} \right) \quad (17)$$

The thermodynamic ideal gas behavior is considered and is defined as followings [37].

$$P_k = \rho_k r T_k, \quad (r = 2.08 \text{ kJ/kg K}) \quad (18)$$

Two assumptions are used by Nlog code depending on the wall-type of each Stirling engine chambers. The assumptions for an isentropic regenerative type wall are shown in Eqs. (19.a) and (19.b) represents the assumption for the isothermal type.

$$\frac{dQ_{e_k}}{dt} = 0 \quad (19.a)$$

$$\frac{dT_{w_k}}{dt} = 0 \quad (19.b)$$

The formulations of Reynolds number (Re) and kinetic Reynolds number (Re_w) are described in Eqs. (20)–(23).

$$Re_k = \frac{\rho_k v_k D_{h_k}}{\mu_k} \quad (20)$$

$$Re_{w_k} = \frac{\rho_k D_{h_k}^2 \dot{\theta}}{\mu_k} \quad (21)$$

$$\mu_k = 1.475 \times 10^{-6} \frac{T_k^{1.5}}{T_k + 80} \quad (22)$$

$$D_{h_k} = \frac{4A_{f_k}}{A_{w_k}} \quad (23)$$

A schematic of the engine linkage kinematic parameters that are modeled by the code is shown in Fig. 3.

$$b_1 = \sqrt{L^2 - (e - r)^2} \quad (24)$$

$$b_2 = \sqrt{(L - r)^2 - e^2} \quad (25)$$

$$b_\theta = \sqrt{L^2 - (e + r)^2} \quad (26)$$

$$V_e = V_{cle} + A_d(b_\theta - b_2 - r \sin \theta) \quad (27)$$

$$V_c = V_{clc} + 2A_p(b_1 - b_\theta) \quad (28)$$

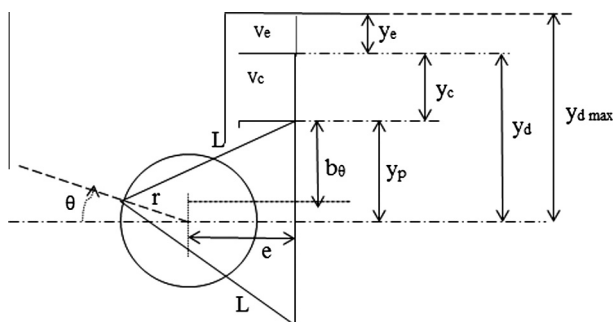


Fig. 3. GPU3 linkage kinematics.

Thus, V_e to V_c could be expressed as the functions of crank angle according to Eqs. (27) and (28) of which the parameters b_1 , b_2 , and b_θ are obtained from Eqs. (24)–(26), respectively. For each control volume which is characterized by k index, Eqs. (11)–(18), contain 12 unknown variables including x , τ , Nu , v_f , ρ , P , u , Q_e , Q_c , K , T_w and T . The values of τ and Nu are expressed as the functions of Reynolds number Re and kinetic Reynolds number Re_w according to the experimental correlations which have been presented by Kays and London [39], de Monte et al. [40,41] and Zheng [42] for different ranges of Re , Re_w different types and dimensions of cross sectional area, the wetted surface roughness can be obtained for the engine heat exchangers.

Eqs. (11)–(19) totally form a system of nine equations which are solved by Nlog code to determine the nine remained unknown variables at each time step for each control volume. Finally, the code running results including the engine output power and the rejected heat are calculated using Eqs. (29) and (30).

$$p = \sum_{k: \text{Variable volumes}} A_{f_k} P_k \frac{dx_k}{dt} \quad (29)$$

$$\dot{Q}_r = \sum_{k: \text{Cooler volumes}} \dot{Q}_{e_k} \quad (30)$$

6. Results and discussion

For the purpose of initial investigation on the final model for determining the objective and criteria, a preliminary study and modeling was carried out, which is firstly described as below.

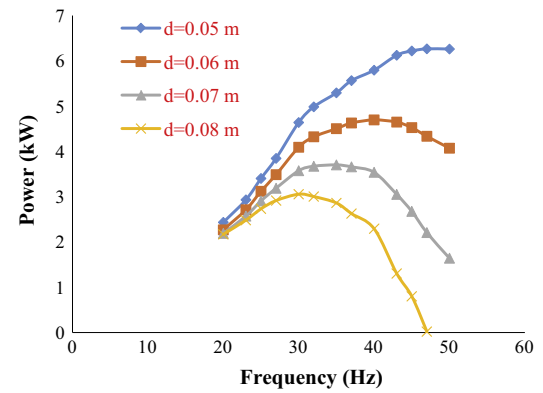


Fig. 4. Variation of the output power of the Stirling engine for different frequency at different strokes.

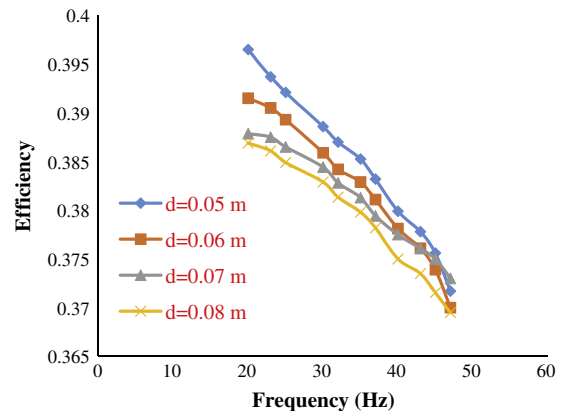


Fig. 5. Variation of the efficiency of the Stirling engine for different frequency at different strokes.

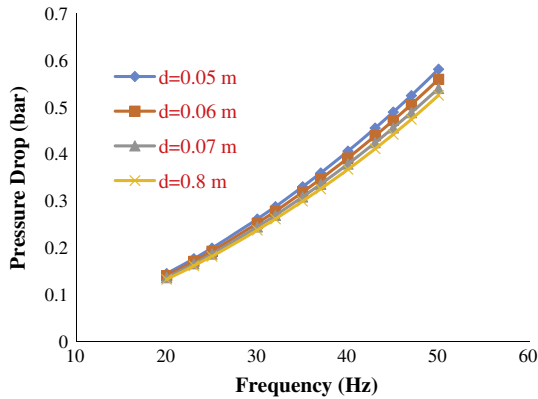


Fig. 6. Variation of the pressure drop of the Stirling engine for different frequency at different strokes.

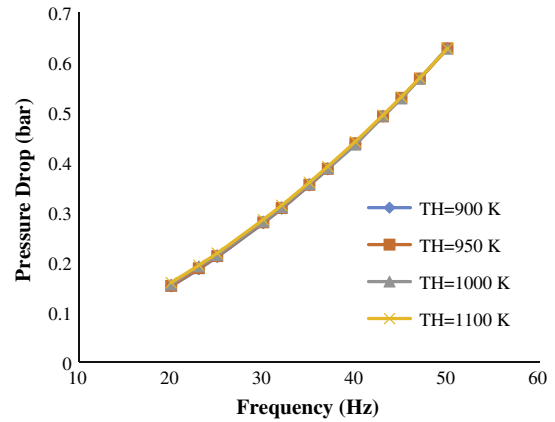


Fig. 9. Variation of the pressure drop of the Stirling engine for different frequency at different temperatures.

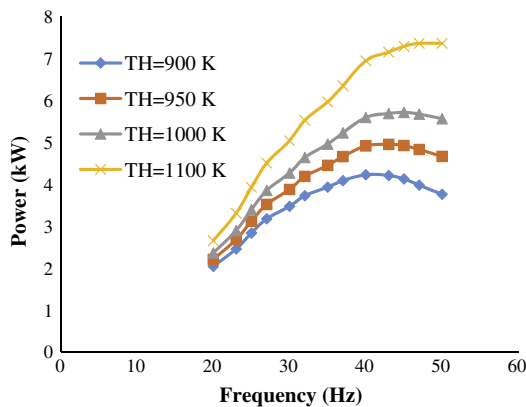


Fig. 7. Variation of the output power of the Stirling engine for different frequency at different 515 temperatures.

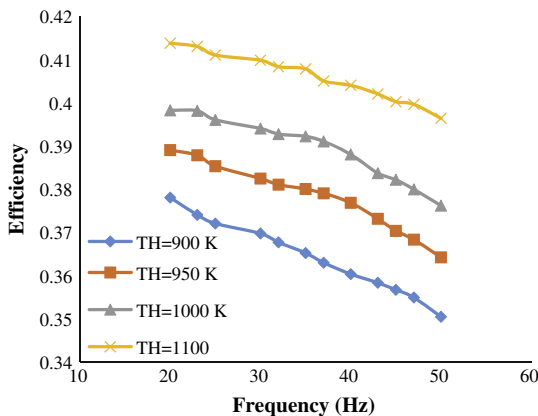


Fig. 8. Variation of the efficiency of the Stirling engine for different frequency at different temperatures.

6.1. The effects of frequency, stroke and heat source temperature

According to Fig. 3, the output power is increased up to a certain point by increasing frequency, after that, the power is decreased with increasing frequency. Also in certain frequency, with increasing the stroke, the output power is decreased. As it is demonstrated in Fig. 4, by increasing the frequency, the efficiency is decreased. Because, by increasing frequency, the working fluid has less time

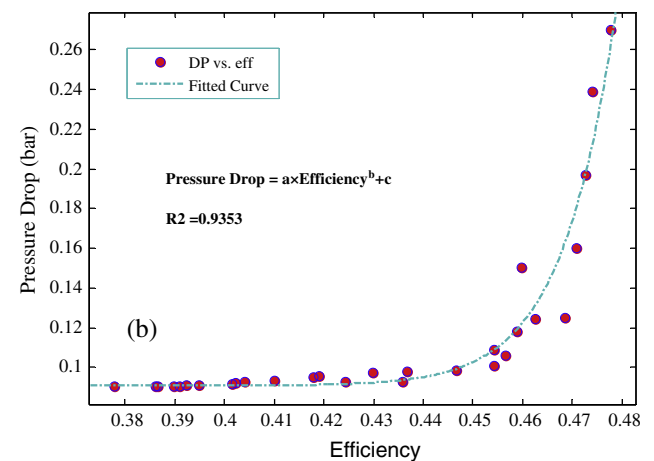
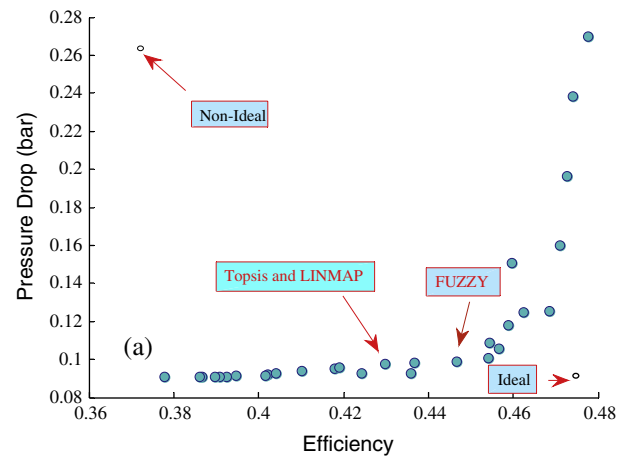


Fig. 10. (a,b) Pareto optimal Frontier in the objectives space (pressure drop-efficiency).

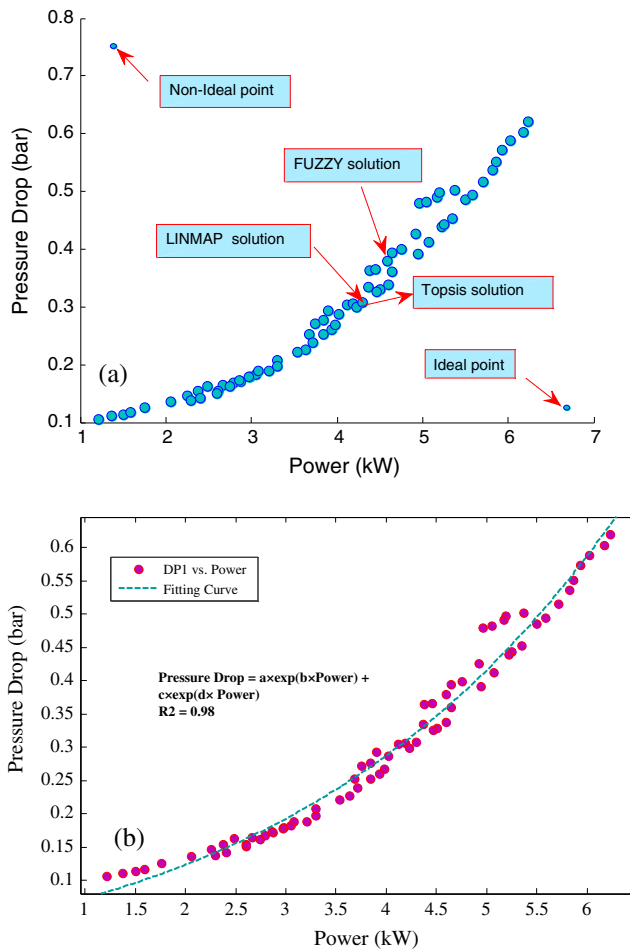
for heat exchanging between the hot source and the cold source. As it is shown in Fig. 5, by increasing frequency, the differential pressure drop is increased. It is happened due to creating pressure drop for increasing the engine fluctuations. All these results are reported for a constant temperature (see Fig 6).

The effect of heat source temperature on the output power in different frequency is demonstrated in Fig. 7. As can be seen in Fig. 7, by changing the frequency, the output power has a maximum power, and until a certain point the power is increased then decreased. Also in a certain frequency, by increasing heat source

Table 2

The coefficients and goodness of fit for pressure drop–efficiency curve.

Coefficient			
$a = 3.629\text{e}+13$	$b = 44.65$		$c = 0.09088$
Goodness of fit			
SSE: 0.003777	R-square: 0.9353	Adjusted R-square: 0.9304	RMSE: 0.01205

**Fig. 11.** (a,b) Pareto optimal Frontier in the objectives space (pressure drop–power).

temperature, the output power is increased. According to Fig. 8, the efficiency is decreased by increasing frequency, because the differential pressure drop is increased by increasing frequency (Fig. 9) and the working fluid has less time for heat exchanging between the hot source and the cold source. The results of this section are presented at constant stroke, 0.03, which is taken from the experimental work.

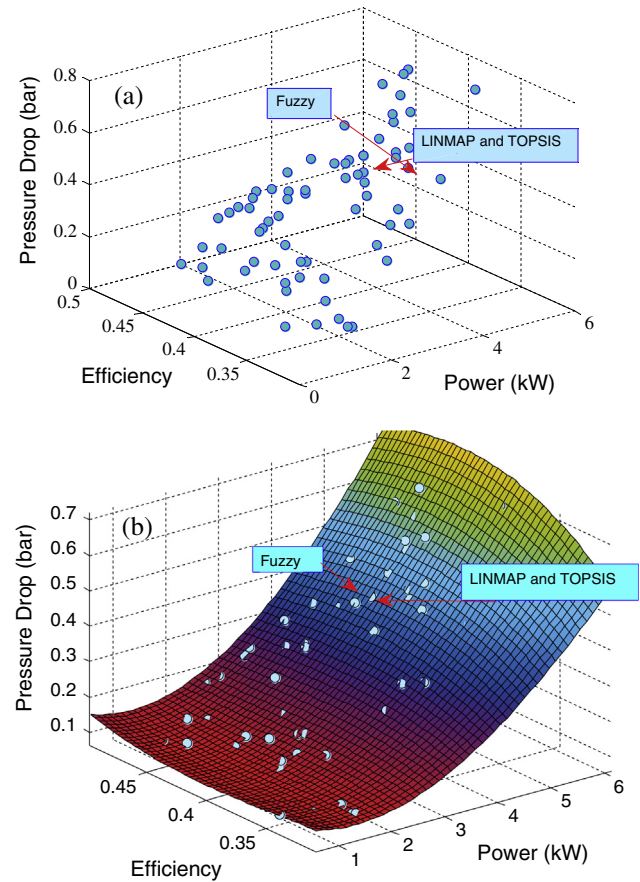
6.2. The results of optimization

For reaching the aim of the multi-objective optimization, the pressure drop (bar) of the Stirling engine must be minimized and the efficiency and the output power (kW) of the Stirling engine

Table 3

The coefficients and goodness of fit for pressure drop–power curve.

Coefficient			
$a = -53.46$	$b = 0.2064$	$c = 53.49$	$d = 0.2068$
Goodness of fit			
SSE: 0.02932	R-square: 0.98	Adjusted R-square: 0.9791	RMSE: 0.02108

**Fig. 12.** (a and b) Pareto optimal Frontier in the objectives space (pressure drop–power–efficiency).

must be maximized. The optimization was carried out using a class of multi-objective evolutionary algorithm called NSGA-II. The optimization process was performed using the third order thermodynamic analysis with the constraints which were expressed in (2)–(5). Fig. 10a illustrates the Pareto Frontier in the space for pressure drop and efficiency. Three final solutions were selected by the Fuzzy, LINMAP and TOPSIS decision-making methods which are indicated in this figure. In Fig. 10a, it is shown that the optimal values of the differential pressure are varied from 0.08 bar to 0.28 bar and the optimal value of the efficiency is between 0.38 and 0.48. The equation of the Pareto optimal Frontier is obtained and indicated in Fig. 10b, which can be applied to gain the system pressure drop, as followings:

$$dP = a \times \eta^b + c \quad (31)$$

Table 4

The coefficients and goodness of fit for pressure drop-power-efficiency curve.

<i>Coefficient</i>				
$p_{00} = -0.09471$	$p_{10} = -0.4763$	$p_{01} = 3.673$	$p_{20} = 0.02009$	$p_{11} = 2.182$
$p_{02} = -14.24$	$p_{30} = 0.001209$	$p_{21} = 0.03438$	$p_{12} = -2.848$	$p_{03} = 16.14$
<i>Goodness of fit</i>				
SSE: 0.01437	R-square: 0.992	Adjusted R-square: 0.9908		RMSE: 0.01574

Table 5

Comparison of the final optimal solutions of efficiency-pressure drop (multi objective-multi variable) with experimental results.

Methods	Decision variables				Objective functions	
	T_H (K)	d (m)	P (bar)	f (Hz)	Efficiency (%)	Pressure drop (bar)
Initial theoretical status	977	0.03	41.3	47.1	36.17	0.475
TOPSIS	90187	0.0635	21.25	21.17	43	0.097
LINMAP	90,187	0.0635	21.25	21.17	43	0.097
Fuzzy	902.96	0.0631	21.69	21.34	45	0.1007
Experiment [43]	977	0.03	41.3	47.1	35	–

Table 6

Comparison of the final optimal solutions of power-pressure drop (multi objective-multi variable) with experimental results.

Methods	Decision variables				Objective functions	
	T_H (K)	d (m)	P (bar)	f (Hz)	Power (kW)	Pressure drop (bar)
Initial theoretical status	977	0.03	41.3	47.1	4.17	0.475
TOPSIS	1085.87	0.0273	50	32	4.316	0.297
LINMAP	1083.87	0.0243	50	31.73	4.46	0.325
Fuzzy	1022.89	0.0253	48.1	3626	4.7	0.3988
Experiment [43]	977	0.03	41.3	47.1	3.9	–

Table 7

Comparison of the final optimal solutions of power-efficiency-pressure drop (multi objective-multi variable) with experiment results.

Methods	Decision variables				Objective functions		
	T_H (K)	d (m)	P (bar)	f (Hz)	Power (kW)	Efficiency	Pressure drop (bar)
Initial theoretical status	977	0.03	41.3	47.1	4.17	36.17	0.475
TOPSIS	1000.59	0.029	48.36	34.93	4.26	42	0.358
LINMAP	1000.59	0.029	48.36	34.93	4.26	42	0.358
Fuzzy	1022.62	0.045	47.1	41.66	4.51	40.7	0.447
Experiment [43]	977	0.03	41.3	47.1	3.9	35	–

Table 8

Comparison of the final optimal solutions of this paper (multi objective-multi variable) with corresponding results.

Type of model	f (Hz)	P (bar)	d (m)	T_H (K)	Power (Kw)	Efficiency (%)
Initial theoretical model	47.1	41.3	0.03	977	4.17	36.17
LINMAP and TOP SIS	34.93	48.36	0.029	1000.59	4.26	42
Fuzzy	41.66	47.1	0.045	1022.62	4.51	40.7
Urieli and Berchowitz (adiabatic model) [44]	47.1	41.3	0.03	977	8.3	62.5
Timoumi dynamic model without losses [43]	47.1	41.3	0.03	977	7.1	54.96
Timoumi dynamic model with loss dissipation (M1) [45]	47.1	41.3	0.03	977	7.4	53.1
Urieli and Berchowitz quasi-steady flow (pressure drop included) [46]	47.1	41.3	0.03	977	6.7	52.5
Timoumi dynamic best model [46]	47.1	41.3	0.03	977	4.2	38.49
Experimental [43]	47.1	41.3	0.03	977	3.9	35
Error between initial theoretical model and experimental	–	–	–	–	6.9	3.3

The italic values of f - frequency; d - diameter; T_H - hot side temperature.

The coefficients (with 95% confidence bounds) and goodness of fit are summarized for the pressure drop and the efficiency in Table 2.

The Pareto optimal Frontier for the output power and pressure drop is illustrated in Fig. 11a, that is a two objective function. It is clear that the optimal value of the differential pressure is varied from 0.1 bar to 0.68 bar, and the optimal value of the power is between 1 and 6.5 kW. The equation of the Pareto optimal Frontier is obtained and indicated in Fig. 11b. The coefficients (with 95% confidence bounds) and goodness of fit are summarized for the pressure drop and the efficiency in Table 3. As it can be seen, if

the objective functions would be power and pressure drop, the heat source temperatures for engine using LINMAP, TOPSIS, and FUZZY methods will be 1083.78, 1085.35, and 1022.894, respectively. Also, the strokes are obtained as 0.0273, 0.0243, 0.0253 (m) using LINMAP, TOPSIS, and FUZZY methods, respectively.

$$dP = a \times \exp(b \times p) + c \times \exp(d \times p) \quad (32)$$

Pareto optimal Frontier for three objective functions, power, and efficiency of Stirling and system pressure drop are shown in Fig. 12a. The optimal results for the decision parameters and

objective functions using LINMAP, TOPSIS and Fuzzy decision-making methods are summarized in Table 4.

The equation of the Pareto optimal Frontier is obtained and indicated in Fig. 12b, which can be applied to gain the system pressure losses, as followings:

$$\begin{aligned} dp = & p00 + p10 \times p + p01 \times \eta + p20 \times p^2 + p11 \times p \times \eta \\ & + p02 \times \eta^2 + p30 \times pr^3 + p21 \times p^2 \times \eta + p12 \\ & \times power \times \eta^2 + p03 \times \eta^3 \end{aligned} \quad (33)$$

where p is output power.

The comparison of the optimal results obtained in this paper using three decision making methods with the experimental results of GPU-3 Stirling engine is demonstrated in Tables 5 and 6. As it can be seen in Table 7, the output power and the efficiency of the engine are increased to 600 W and 5.7%, respectively, when using Fuzzy decision making. Also by the use of TOPSIS and LINMAP decision making, the output power and the efficiency of the engine are increased to 460 W and 7%, respectively.

According to Table 8, the characteristics of this work are compared to the other works which have not considered the important losses. This reason caused that they reported higher efficiency.

7. Conclusions

In this study, the third order thermodynamic analysis has been considered to optimize the output power, the efficiency and the pressure drop of a Stirling engine. The efficiency-pressure drop, the power-pressure drop and the efficiency-power-pressure drop, of the Stirling engine were applied simultaneously for multi-objective optimization and the temperature of heat source, the stroke, the engine pressure charge, and the engine frequency were considered as the design parameters. It was observed that the Fuzzy decision-making method yields in a final solution with more desirable output power and efficiency. Also, in aspect of pressure drop, the TOPSIS and LINMAP with the same amount have a better situation than the Fuzzy model.

Acknowledgement

The authors would like to appreciate the helps and technical supports of Mr. Mazdak Houshang in this study.

References

- [1] Karabulut H, Yücesu HS, Çınar C, Aksoy F. An experimental study on the development of a β -type Stirling engine for low and moderate temperature heat sources. *Appl Energy* 2009;86.1:68–73.
- [2] Ruelas J, Velazquez N, Cerezo J. A mathematical model to develop a Scheffler-type solar concentrator coupled with a Stirling engine. *Appl Energy* 2013;101:253–60.
- [3] Kolin I. *Stirling motor: history-theory-practice*. Dubrovnik: Inter University Center; 1991.
- [4] Dyson RW, Wilson SD, Tew RC. Review of computational Stirling analysis methods. In: *Proceedings of 2nd international energy conversion engineering conference*. Paper No AIAA20045582. Providence RI; 2004. p. 1–21.
- [5] Finkelstein T. Thermodynamic analysis of Stirling engines. *J Spacecraft Rockets* 1967;4(9):1184–9.
- [6] Andersen SK. Numerical simulation of cyclic thermodynamic processes. PhD Thesis, Department of Mechanical Engineering, Technical University of Denmark; 2006.
- [7] Feuer B. Degrees of freedom in the layout of Stirling engines. *Stirling Engines* 1973;18–37.
- [8] Carlsen H; 1991. <www.brad.ac.uk/staff/vtoropov/burgeon/b_stirl.htm>.
- [9] Chen CL, Chia-En H, Yau HT. Performance analysis and optimization of a solar powered Stirling engine with heat transfer considerations. *Energies* 2012;5(9):3573–85.
- [10] Kraitong K, Mahkamov K. Optimization of low temperature difference solar Stirling engines using genetic algorithm. In: *World renewable energy congress*, Sweden; 2011.
- [11] Özyer T, Zhang M, Alhaji R. Integrating multi-objective genetic algorithm based clustering and data partitioning for skyline computation. *Appl Intell* 2011;35(1):110–22.
- [12] Ombuki B, Ross BJ, Hanshar F. Multi-objective genetic algorithms for vehicle routing problem with time windows. *Appl Intell* 2006;24(1):17–30.
- [13] Blecic I, Cecchini A, Trunfio GA. A decision support tool coupling a causal model and a multi-objective genetic algorithm. *Appl Intell* 2007;26(2):125–37.
- [14] Van Veldhuizen DA, Lamont GB. Multi-objective evolutionary algorithms: analyzing the state-of-the-art. *Evolut Comput* 2000;8(2):125–47.
- [15] Konak A, Coit DW, Smith AE. Multi-objective optimization using genetic algorithms: a tutorial. *Reliab Eng Syst Safety* 2006;91(9):992–1007.
- [16] Ahmadi MH, Sayyadi H, Mohammadi AH, Barranco MA. Thermo-economic multi-objective optimization of solar dish-Stirling engine by implementing evolutionary algorithm. *Energy Convers Manage* 2013;73:370–80.
- [17] Ahmadi MH, Sayyadi H, Dehghani S, Hosseinzadeh H. Designing a solar powered Stirling heat engine based on multiple criteria: maximized thermal efficiency and power. *Energy Convers Manage* 2013;75:282–91.
- [18] Lazzaretto A, Toffolo A. Energy, economy and environment as objectives in multi-criterion optimization of thermal systems design. *Energy* 2004;29(8):1139–57.
- [19] Zhang J. Multi-objective shape optimization of helico-axial multiphase pump impeller based on NSGA-II and ANN. *Energy Convers Manage* 2011;52(1):538–46.
- [20] Toffolo A, Lazzaretto A. Evolutionary algorithms for multi-objective energetic and economic optimization in thermal system design. *Energy* 2002;27(6):549–67.
- [21] Ahmadi MH, Hosseinzade H, Sayyadi H, Mohammadi Amir H, Kimiaghani F. Application of the multi-objective optimization method for designing a powered Stirling heat engine: design with maximized power, thermal efficiency and minimized pressure loss. *Renew Energy* 2013;60:313–22.
- [22] Martini WR. Stirling engine design manual. No. DOE/NASA/3152-78/1; NASA-CR-135382. Washington Univ., Richland (USA). Joint Center for Graduate Study; 1978.
- [23] Mahmoodi M, Ziabasharhagh M. Numerical solution of beta-type Stirling engine by optimizing heat regenerator for increasing output power and efficiency. *J Basic Appl Sci Res* 2012;2.2:1395–406.
- [24] Cheng CH, Yang HS. Optimization of geometrical parameters for Stirling engines based on theoretical analysis. *Appl Energy* 2012;92:395–405.
- [25] Batmaz I, Üstün S. Design and manufacturing of a V-type Stirling engine with double heaters. *Appl Energy* 2008;85.11:1041–9.
- [26] Cheng C H, Yang HS, Zhou BY, Chen YC, Wang YJ. Dynamic simulation of thermal-lag Stirling engines. *Appl Energy* 2013;108:466–76.
- [27] Solmaz H, Halit K. Performance comparison of a novel configuration of beta-type Stirling engines with rhombic drive engine. *Energy Convers Manage* 2014;78:627–33.
- [28] Cinar C, Yucesu S, Topgul T, Okur M. Beta-type Stirling engine operating at atmospheric pressure. *Appl Energy* 2005;81.4:351–7.
- [29] Calderon V. Multi-objective optimization approach for land use allocation based on water quality criteria. *Dissert Abstr Int* 2010;71(4).
- [30] Deb K. Multi-objective optimization. *Multi-object Optimiz Evolut Algorithms* 2001:13–46.
- [31] Alves MJ, Climaco J. A review of interactive methods for multi objective integer and mixed-integer programming. *Eur J Oper Res* 2007;180.1:99–115.
- [32] Deb K, Agrawal S, Pratap A, Meyarivan T. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Trans Evolut Comput* 2002;6(2):182–97.
- [33] Sayyadi H, Babaelahi M. Multi-objective optimization of a joule cycle for reliquefaction of the liquefied natural gas. *Appl Energy* 2011;88(9):3012–21.
- [34] Back T, Fogel DB, Michalewicz Z. *Handbook of evolutionary computation*. IOP Publishing Ltd.; 1997.
- [35] Sayyadi H, Mehrabipour R. Efficiency enhancement of a gas turbine cycle using an optimized tubular recuperative heat exchanger. *Energy* 2012;38(1):362–75.
- [36] <http://www.stirling.blogsky.com>.
- [37] Robert ATM, Fox W, Pritchard PJ. *Introduction to fluid mechanics*. John Wiley & Sons; 2004.
- [38] Petersen H. The properties of helium: density, specific heats, viscosity and thermal conductivity at pressures from 1 to 100 bar and from room temperature to about 1800 K; 1970.
- [39] Kays WM, London AL. *Compact heat exchangers*. McGraw Hill; 1984.
- [40] de Monte GGF, Marcotullio F. An analytical oscillating-flow thermal analysis of the heat exchangers and regenerator in Stirling machines. In: *Energy conversion engineering intersociety conference*; 1996. p. 1421–7.
- [41] de Monte F. Thermal analysis of the heat exchangers and regenerator in Stirling cycle machines. *J Propul Power* 1997;404–11.
- [42] Shan YZ. Oscillatory flow and heat transfer in Stirling engine regenerator; 1993.
- [43] Timoumi Y, Tlili I, Ben Nasrallah S. Design and performance optimization of GPU-3 Stirling engines. *Energy* 2008;33(7):1100–14.
- [44] Urieli I, Berchowitz DM. Stirling cycle engine analysis. A. Hilger; 1984.
- [45] Toghyani S, Kasaean A, Ahmadi MH. Multi-objective optimization of Stirling engine using non-ideal adiabatic method. *Energy Convers Manage* 2014;80:54–62.
- [46] Formosa, Fabien, Ghislain Despesse. Analytical model for Stirling cycle machine design. *Energy Convers Manage* 2010;51(10):1855–63.